A very commendable feature of the whole treatise is the large number of exercises, which are either original, or selected from many well-known standard works and monographs. Among them are problems of considerable importance in real projective geometry, which by some authors are included in the main subject matter.

The mechanical make up of the two books is excellent, with the exception of many figures, which, from the standpoint of a connoisseur of graphic arts, are by no means the product of expert draftsmen. Pleasing figures, however, are as much to be desired as pleasing type. In this respect the second volume shows a decided improvement over the first.

May be it is a whim of an æsthetic crank if he misses the same kind of imprint on the backs of the two volumes. Why is that of the first volume gilt, that of the second volume black? But we shall not insist further upon such trivialities, and close this review by thinking with Voltaire: Le secret d'ennuyer est celui de tout dire.

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An Elementary Treatise on Curve Tracing. By P. FROST. Fourth edition, revised by R. J. T. BELL. London, Macmillan, 1918. 8vo. 16 + 210 pp. + 17 plates. Price 12s. 6d.

The first edition of this work appeared in 1872 and reprints were issued as second and third editions in 1892 and 1911 respectively. Concerning the present edition (which is the first revision of the original work) Mr. Bell states that it does not differ from the previous one except in places where alterations were necessary to remove ambiguities or to correct mistakes in analysis and diagrams. A useful classified list of the curves discussed has been added on pages 203–208, and the typography of the new edition is much better than that of the earlier one. Except for 7 types of curves whose equations involve trigonometric functions, all curves analyzed have rational algebraic equations; these include 23 cubics, 69 quartics, 31 quintics, 18 sextics, 9 curves of the seventh degree and 2 of the eighth. As the calculus is nowhere employed in the discussion great ingenuity is displayed in the analyses, and the work as a whole is exceedingly interesting.

In his preface of 1872 (reprinted in the present edition) Frost wrote that it would be difficult to find another subject

"which, with a very limited extent of reading, combines, to the same extent, so many valuable hints of methods of calculations to be employed hereafter, with so much pleasure in its use.

For example, the subject of Graphical Calculations is coming more into use every day, and is applied with success to many difficult problems in Statics, Engineering and Crystallography; hints of this the student will find in the practical solution of divers questions and in the determination of the number of their real roots, which are obtained by graphical methods with great facility.

Again, the methods of successive approximations which are employed in Optics and Astronomy are illustrated in the process of finding asymptotes and approximations to the forms of curves at a finite distance.

The comparison of large and small quantities of different orders of magnitude contains the staple of many of the most important applications of Mathematical Analysis; the Lunar and Planetary Theories depending almost entirely upon such considerations of relative magnitude.

The habit of looking towards an infinite distance, and discussing what takes place there, will render less startling a multitude of conceptions having in them a tendency to produce a feeling of vagueness, such, for instance, as the treatment of the mechanical effect of a couple as synonymous with that of an infinitely small force acting at an infinitely great distance.

As an important point, I would mention the tentative character of the inverse problem in which the form of a curve being given, its equation is to be investigated; the kind of uncertainty which will remain on the mind on account of defective estimation of magnitudes; and the necessity of a selection of what may appear the best of many possible solutions; all this will prepare the student for disappointment which, having perhaps a wrong notion of what is meant by calling mathematics an exact science, he will feel in the conflict of theories by which it is attempted to reconcile the results of experiment in such subjects as Heat, Light, Electricity, and Molecular action generally; for an instance of this I may refer to the battle of philosophers about the direction of vibration of the ether in Plane Polarization."

Contents—Chapter I: Introductory theorems; definitions; tracing by points; symmetry, 1–8. II: Orders of small quantities; forms of parabolic curves near the origin; cusps; tangents to curves; curvature, 9–19. III: Forms of parabolic curves at an infinite distance; examples of tracing curves; trigonometrical curves; illustrations of theory of equations; rules for approximation, 20–37. IV: Forms of curves in the neighbourhood of the origin; simple tangents; direction and amount of curvature; multiple points of two branches; curvature of branches at multiple points; multiple points of higher orders, 38–57. V: Forms of branches whose tangents at the origin are the coördinate axes, 58–67. VI: Asymptotes; points of intersection at an infinite distance; asymptotes parallel to the axes, 68–87. VII: Asymptotes not parallel to the axes; asymptotes to homogeneous curves, 88–106. VIII: Curvilinear asymptotes, 107–116. IX: The analytical triangle [Newton's parallelogram and De Gua's triangle]; properties of the analytical triangle, 117–132. X: Singular points; division into compartments; special curve of the fourth degree, 133–166. XI: Systematic tracing of curves; repeating curves, 167–185. XII: Inverse process; determination of the equation of a given curve, 186–202.

Matrices and Determinoids. By C. E. CULLIS. Cambridge: at the University Press. Royal 8vo. Volume 1, 1913; 12 + 430 pp. Price 21 shillings. Volume 2, 1918; 24 + 555 pp. Price 42 shillings.

Extracts from prefaces: "The present work is an amplification of a course of lectures given for the University of Calcutta in the winter of 1909–10. Its chief feature is that it deals with rectangular matrices and determinoids as distinguished from square matrices and determinants, the determinoid of a rectangular matrix being related to it in the same way as a determinant is related to a square matrix. . . . The first volume contains the most fundamental portions of the theory, and concludes with the solution of any system of linear algebraic equations, which is treated as a special case of the solution of a matrix equation of the first degree. . . . The second volume contains those parts of the theory which naturally precede any investigation of the special properties of functional matrices, i. e., matrices whose elements are rational integral functions of a finite number of variables. It deals almost exclusively with matrices whose elements are constants, which may be arbitrary parameters, and with those transformations of such matrices which are classed as equigradent. It does not however contain all the properties of such matrices. There remain many properties which it will be more convenient to consider after a preliminary study of functional matrices. . . . The following is a list of the books which have had most influence on the work as a whole: Bôcher's *Introduction to Higher Algebra*, Heffter and Koehler's *Lehrbuch der Analytischen Geometrie*, Muth's *Elementarteiler*, Netto's *Vorlesungen über Algebra*, Veronese's *Fondamenti di geometria a piu dimensioni*, Whitehead's *Universal Algebra*. My indebtedness to these and other writers will be more easily recognized in those portions of the work, occurring chiefly in volume 3, which are interpolations in the original scheme." It is expected that the third volume will contain the completion of the theory and "applications to vector analysis and the theory of invariants. The complete exposition was in fact undertaken with a view to these last mentioned applications."

Contents—Volume 1, chapter I: Introduction of rectangular matrices and determinoids, pages 1–21; II: Affects of the elements and derived products of a matrix or determinoid, 22–54; III: Se-